Predicting the unpredictable: Is the electrical spot price chaotic?

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Outline

1. Electricity in NZ

2. Performance of predictive models

3. Chaos,
   1. how to find it,
   2. Implications of a chaotic spot price

Notice how we didn’t say that it is chaotic
Sustainable electricity

- Continuous & secure
  - Electricity is inelastic
  - and unstorable

- Without adverse effects on
  - Environment
  - Future generations

- Affordable
- Predictable?
Deregulation

- Since 1996
- Problems
  - Self regulating, governed by NZEM, but
  - Market not run effectively.
  - Cannot agree on a set of rules.
  - Security of supply problems.
  - 1st March 2004 - Electricity Commission

Compare to Nordic Pool
Recent news

- **Maui**
  (ref: Blakeley 2003)

- **Project Aqua** — 524MW capacity.
  Cancelled April, $45 million invested.

- **Transmission limitations.** Electricity Commission’s plan on 4th June, pay to reduce demand in Upper South Island.
Spot price

Typical price 5-10c /kWhr
What causes excessive spot prices?

- We expect affected by:
  - Lake levels
  - Derivative (i.e. inflow)

- But this is not always the case in NZ …

- Pricing signal – not timely.  
  *Infrastructure Stocktake, PwC 2004*

- … Unlike the Nordic Pool.

*So things are not as simple as they might appear*
Hydro level vs. Day in year with Spot Price ($/MWhr) for different ranges:
- 0-75
- 75-150
- >150

Legend:
- Empty
- Full

Axes:
- Spot price
- Time of year
- Hydro level

Graph shows the distribution of spot prices over time and hydro level.
Predictive models

- What can one model?
  - Unit operations: turbines, local climate, aerofoils
  - Countrywide e.g.: transmission
  - Market

- How complex?
  - Blackbox/heuristic
  - Econometric

- Over what time scale?
  - Long term planning (20 years)
  - Medium term (5 years)
  - Days/weeks purchasing
Logistic models

\[ C = \frac{F}{1 + e^{\theta_0 + \theta_1 t}} \]

Bolger & Tay, 1987
US models – over predicted

Various predictions

Actual energy use

Predictions started here

25 years!

Ref: Craig et al, 2002
Econometric models

- Considerably more complicated

Electricity Model

Other models

- Spot price
- Overall Usage
- Domestic/Industrial sectors
- Fuel type consumptions
- Emission
- Capital requirements

“Grey” box model

Model predictions

Lake levels & flowrates
Econometric models

New Zealand Electricity Generation

Year GWh pa

- CAE (94) Total
- CAE (02) Total
- MComm (00) Total
- MComm (94) Total
- Historical Elec Gen GWh

Performance (rel. error)
2002 Figures

CAE (2002) 0%
CAE (1994) -11%
MComm (2000) -5%
MComm (1994) -5%
Chaotic systems

Los Angeles, USA
Chaotic systems

- Chaos is deterministic

- A parsimonious representation of complex behaviour

- Chaos is difficult to define
  - Dynamics must be nonlinear
  - But we don’t require stochastic and/or chaotic inputs to be chaotic

- Sensitive to:
  - Parameters & structure
  - Initial conditions

- How can we tell if it is chaotic?
  - Dominant Lyapunov coefficient, $\lambda > 0$
Sensitivity to initial conditions

A discrete chaotic system: Henon’s attractor

\[
x_{k+1} = y_k - 1.4 \, x_k^2 + 1
\]
\[
y_{k+1} = 0.3 \, x_k
\]

Starting from:

\[
\begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

But suppose we change the starting point *very slightly*

\[
\begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} = \begin{bmatrix} 1.000001 \\ 1.000001 \end{bmatrix}
\]

Very small \( \Delta x \)
Ways to estimate $\lambda$

1. Direct methods
   1. Lots of carefully controlled experiments
   2. Large data sets

2. Jacobian methods
   1. Estimate from the individual Jacobian matrices

*Finding Chaos in Noisy Systems*, Nychka et al, 1992
Implementation

- Fortran code LENNS (from Nychka et al)
- Standard numerical optimisation routines (BFGS)
- Some technical modifications
- Large computation requirements
  - 50 computers @ 2 hours
  - Embarrassingly parallel
- Matlab for analysis
Weak assumptions on the stationarity of $\lambda$

Is there any movement of $\lambda$ with time?
Spot price & the dominant Lyapunov exponent

![Graph showing the Benmore Spot Price and Lyapunov exponent over time. The graph indicates chaotic and stable periods.]

- Chaotic here
- Stable here

Day #
Work in progress

- Relate $\lambda$ to observable trends/outcomes
- Compute $\lambda$ at different frequencies
  - $\frac{1}{2}$ hourly price, 5 min price
  - Is $\lambda = f(\text{sample interval})$?

Technical issues:
1. Time delay
2. Discretisation
3. Steady-state offset?
Consequences of chaos

- Chaotic trends are suboptimal & unsustainable

- Chaotic systems are unpredictable
  - In the medium & long term
  - Emarket [http://www.energylink.co.nz/emarket.htm](http://www.energylink.co.nz/emarket.htm)

- Unpredictable price increases affect industrial production.
  - *Infrastructure stocktake reportback*, M. Cullen, 2004

- The chaos may be broken by
  - Changing structure:
    - Opening feedback loops
  - Changing parameters
Conclusions

- Spot price excursions not simply correlated to climate
- Previous demand models are ...
- Spot price exhibits chaos
  - At times
  - Often before excursions
- So can we break the chaotic characteristics?
Questions ?