Predicting the unpredictable: Is the electrical spot price chaotic?

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Outline

1. Electricity in NZ
2. Performance of predictive models
3. Chaos,
   1. how to find it,
   2. Implications of a chaotic spot price

Notice how we didn't say that it *is* chaotic

Sustainable electricity

- Continuous & secure
  - Electricity is inelastic
  - and unstorable
- Without adverse effects on
  - Environment
  - Future generations
- Affordable
- Predictable?

The grid

Deregulation

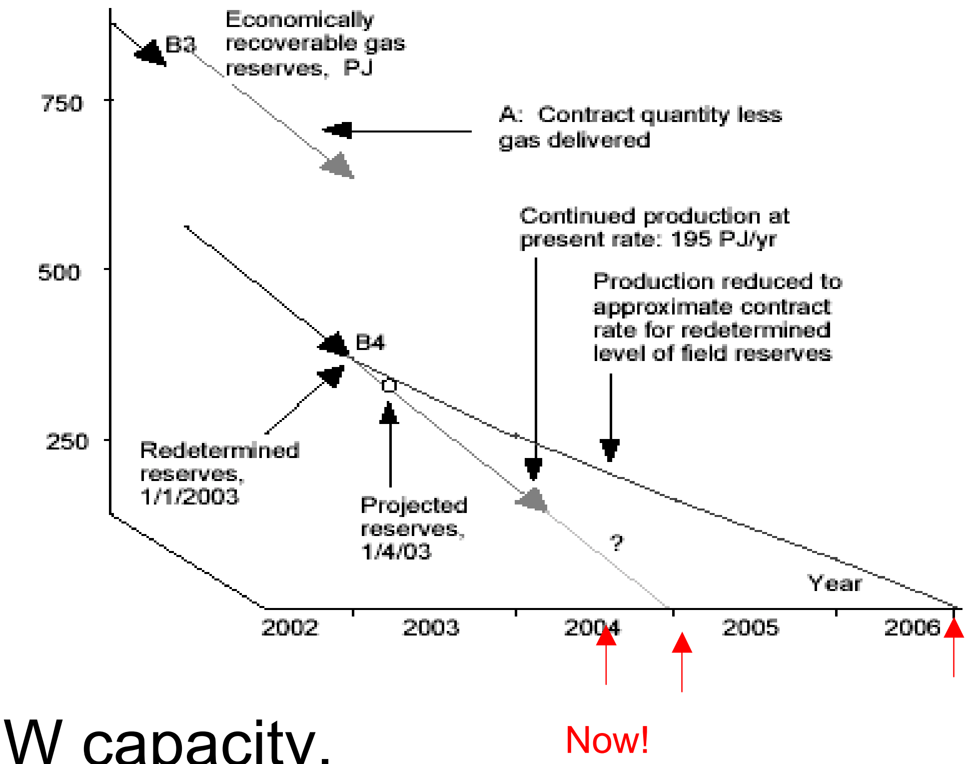
- Since 1996
- Problems
  - Self regulating, governed by NZEM, but
  - Market not run effectively.
  - Cannot agree on a set of rules.
  - Security of supply problems.
  - 1st March 2004 - Electricity Commission

Compare to Nordic Pool

Recent news

- **Maui**

  (ref: Blakeley 2003)

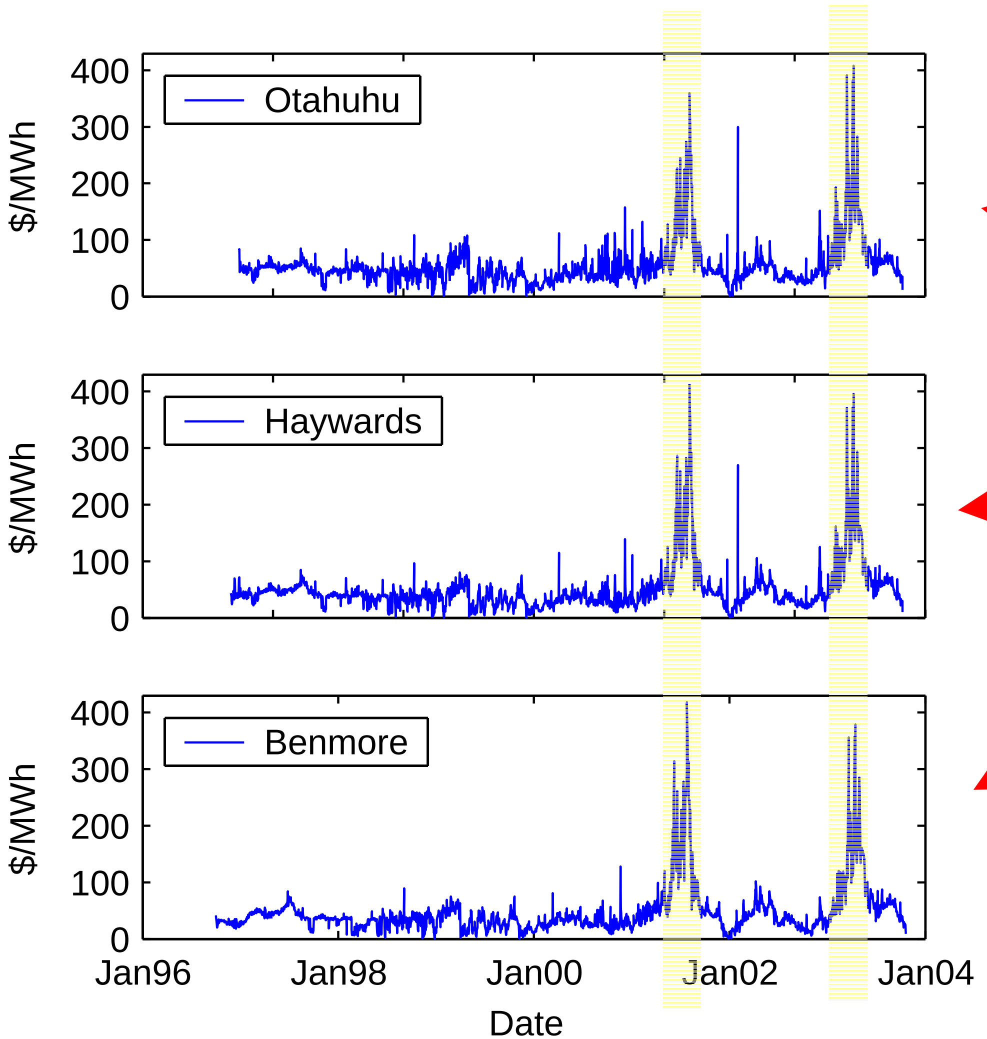


- **Project Aqua** – 524MW capacity.

  Cancelled April, $45 million invested.

- **Transmission limitations**. Electricity Commission's plan on 4th June, pay to reduce demand in Upper South Island.

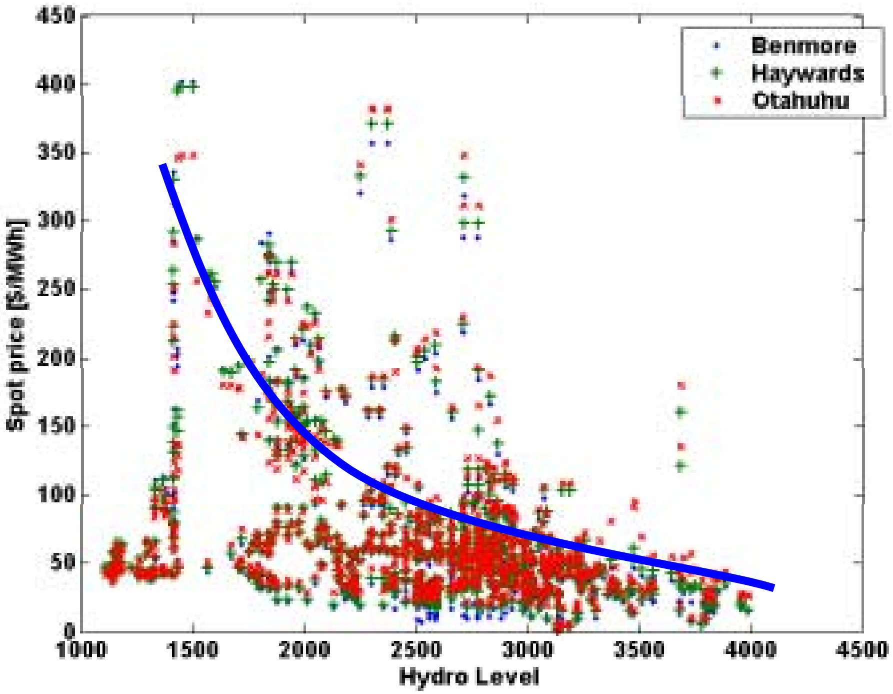
Spot price



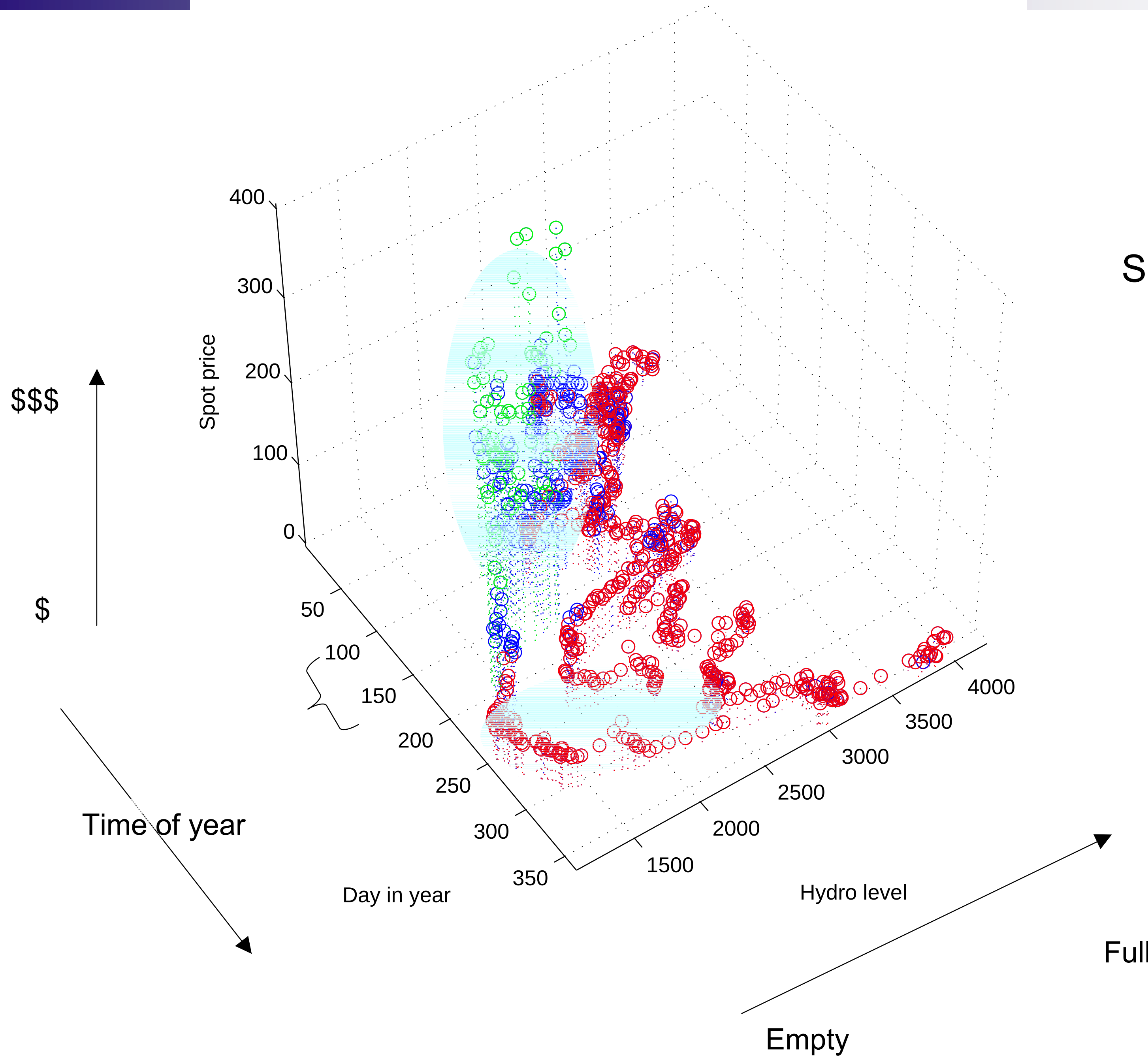
Typical price
5-10c /kWhr

What causes excessive spot prices?

- We expect affected by :
  - Lake levels
  - Derivative (i.e. inflow)
- But this is not always the case in NZ …
- Pricing signal – not timely.
  Infrastructure Stocktake, PwC 2004
- … Unlike the Nordic Pool.



So things are not as simple as they might appear

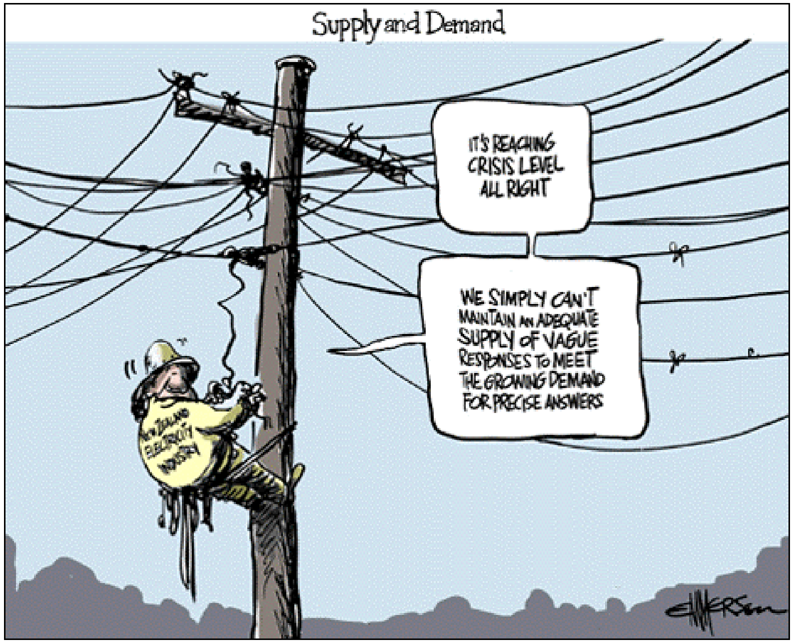


Spot Price
$/MWhr

0-75
75-150
>150

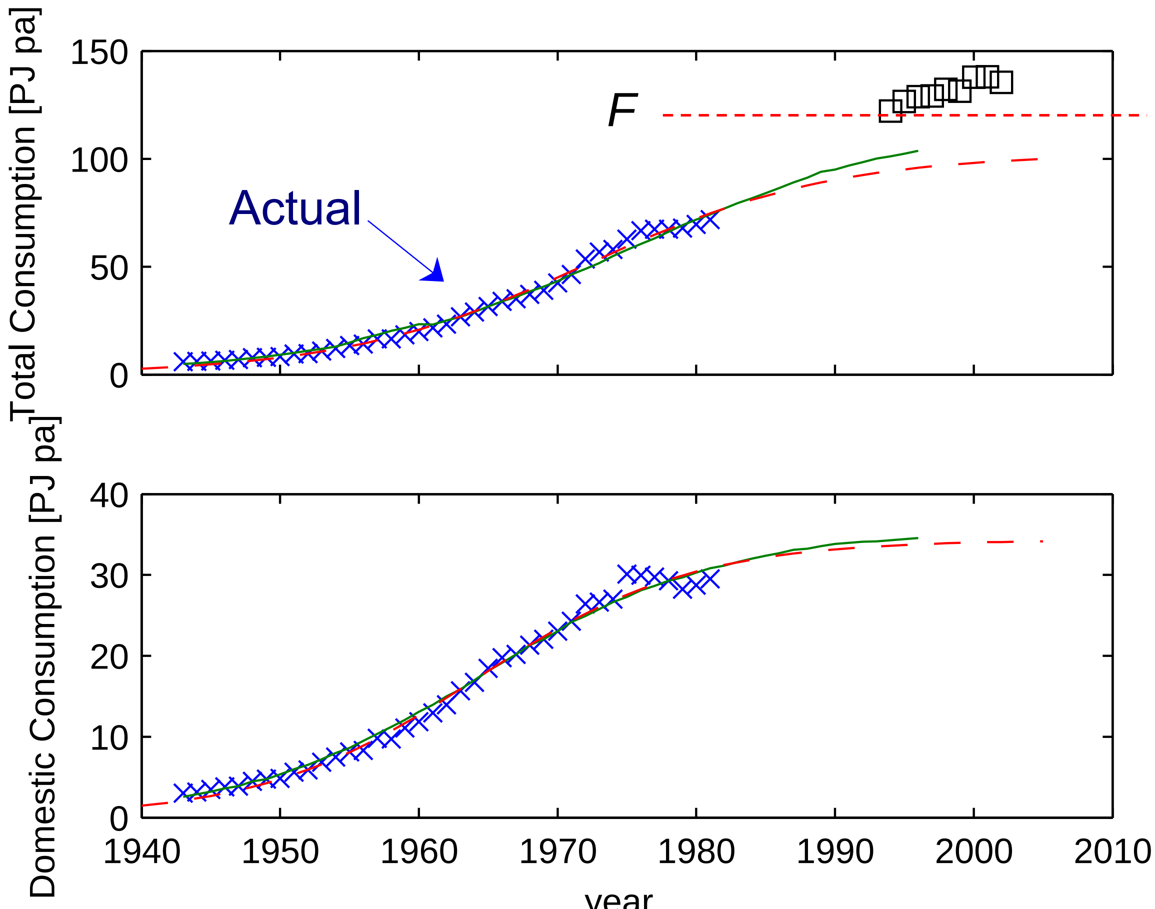
Predictive models

- What can one model?
  - Unit operations: turbines, local climate, aerofoils
  - Countrywide e.g.: transmission
  - Market
- How complex ?
  - Blackbox/heuristic
  - Econometric
- Over what time scale ?
  - Long term planning (20 years)
  - Medium term (5 years)
  - Days/weeks purchasing



Logistic models

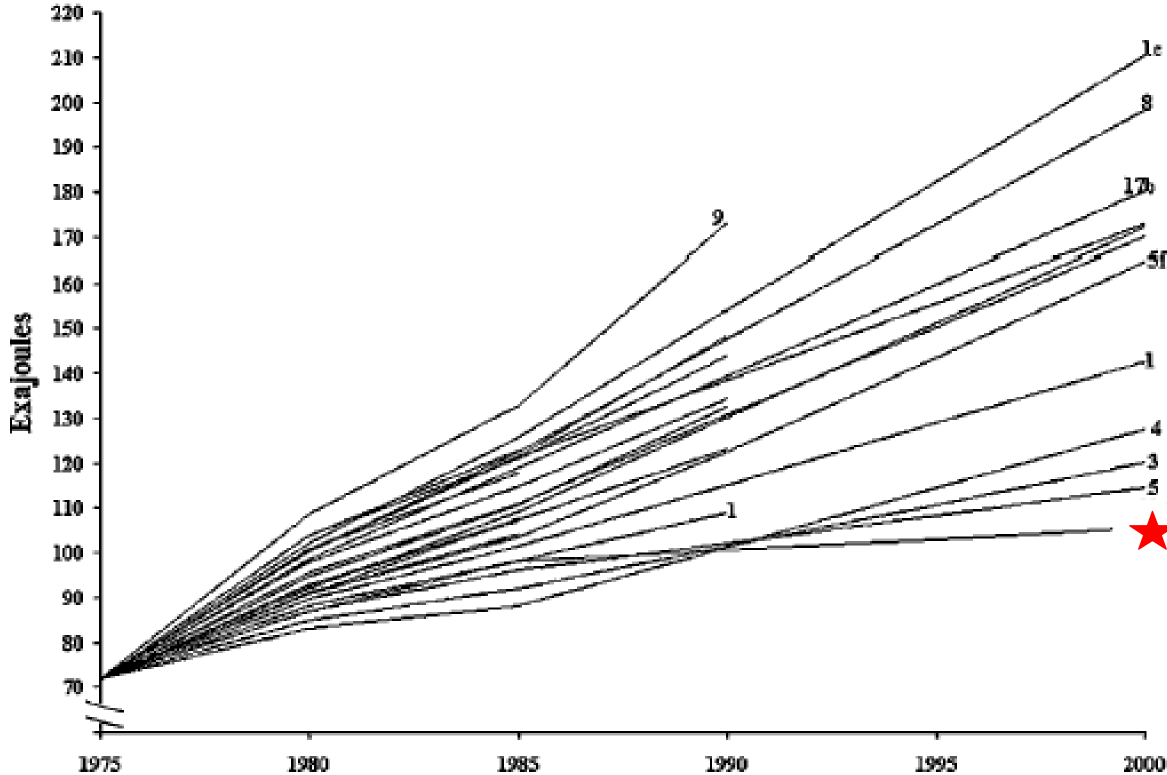
$$C = \frac{F}{1 + e^{\theta_0 + \theta_1 t}}$$



Under predicted In NZ

Bolger & Tay, 1987

US models – over predicted



Various predictions

Actual energy use

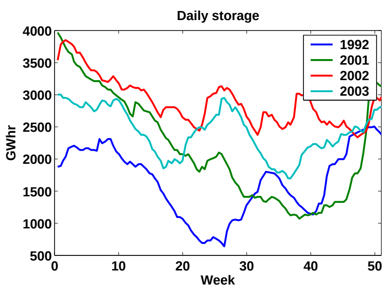
25 years!

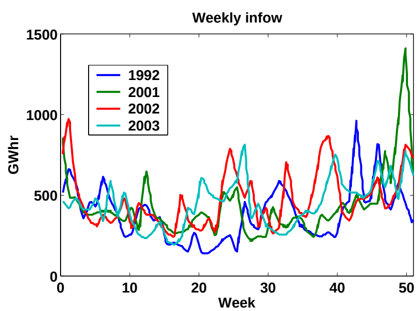
Predictions started here

Ref: Craig *et al*, 2002

Econometric models

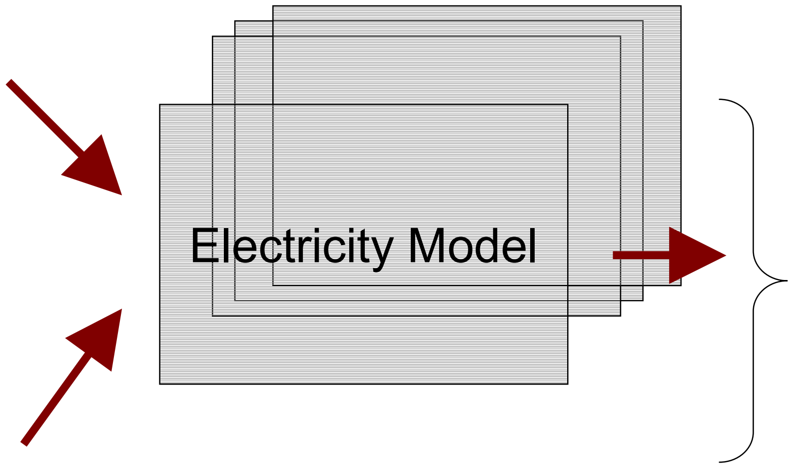
- Considerably more complicated





Lake levels & flowrates

Other models

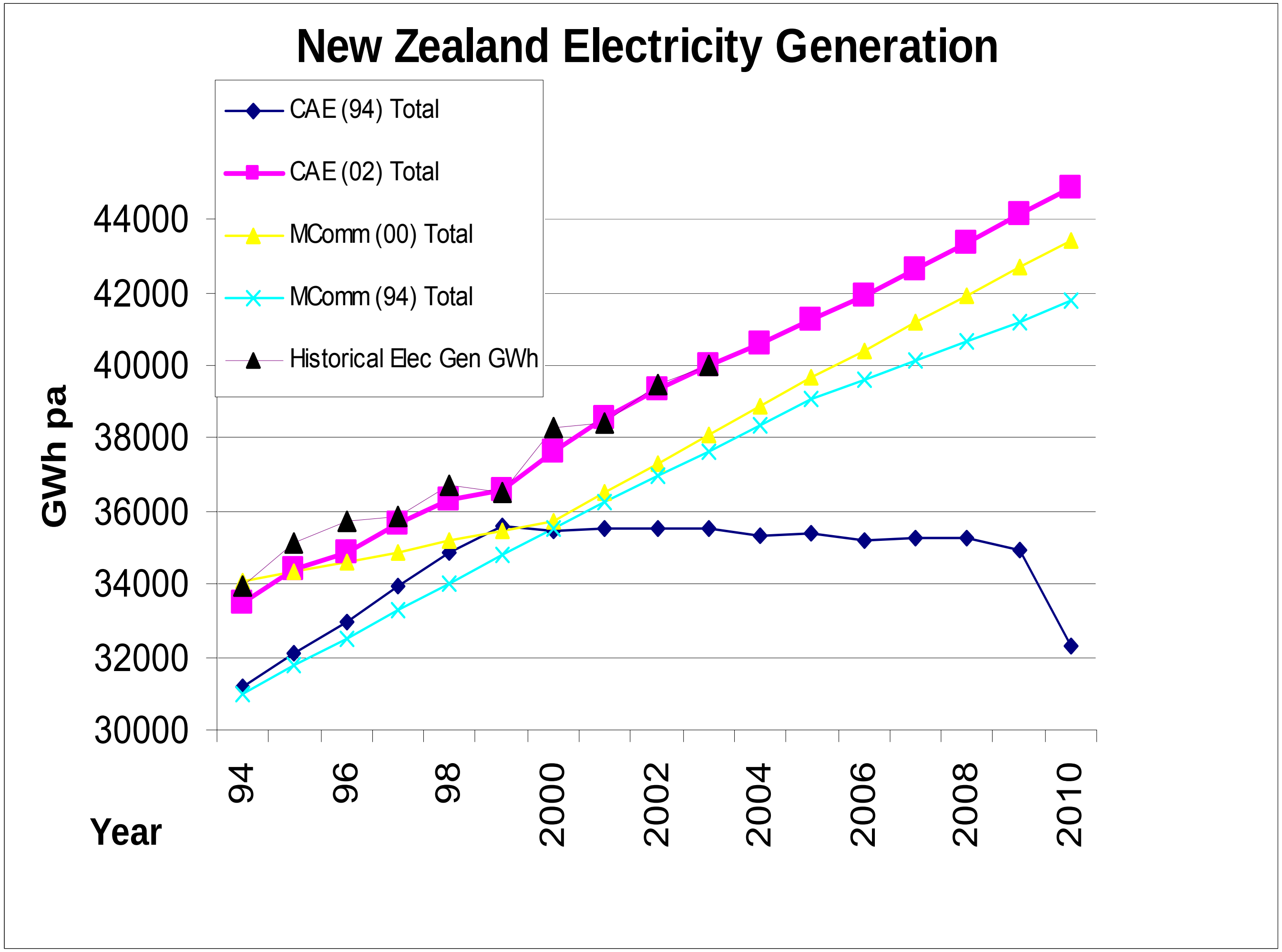


"Grey" box model

- Spot price
- Overall Usage
- Domestic/Industrial sectors
- Fuel type consumptions
- Emission
- Capital requirements

Model predictions

Econometric models



Performance (rel. error)
2002 Figures

CAE (2002) 0%
CAE (1994) -11%
MComm (2000) -5%
MComm (1994) -5%

Chaotic systems

Los Angeles, USA

Chaotic systems

- Chaos is deterministic

- A parsimonious representation of complex behaviour

- Chaos is difficult to define
  - Dynamics must be nonlinear
  - But we don’t require stochastic and/or chaotic inputs to be chaotic

- Sensitive to:
  - Parameters & structure
  - Initial conditions

- How can we tell if it is chaotic?
  - Dominant Lyapunov coefficient, $\lambda>0$

Sensitivity to initial conditions

A discrete chaotic system:
Henon's attractor

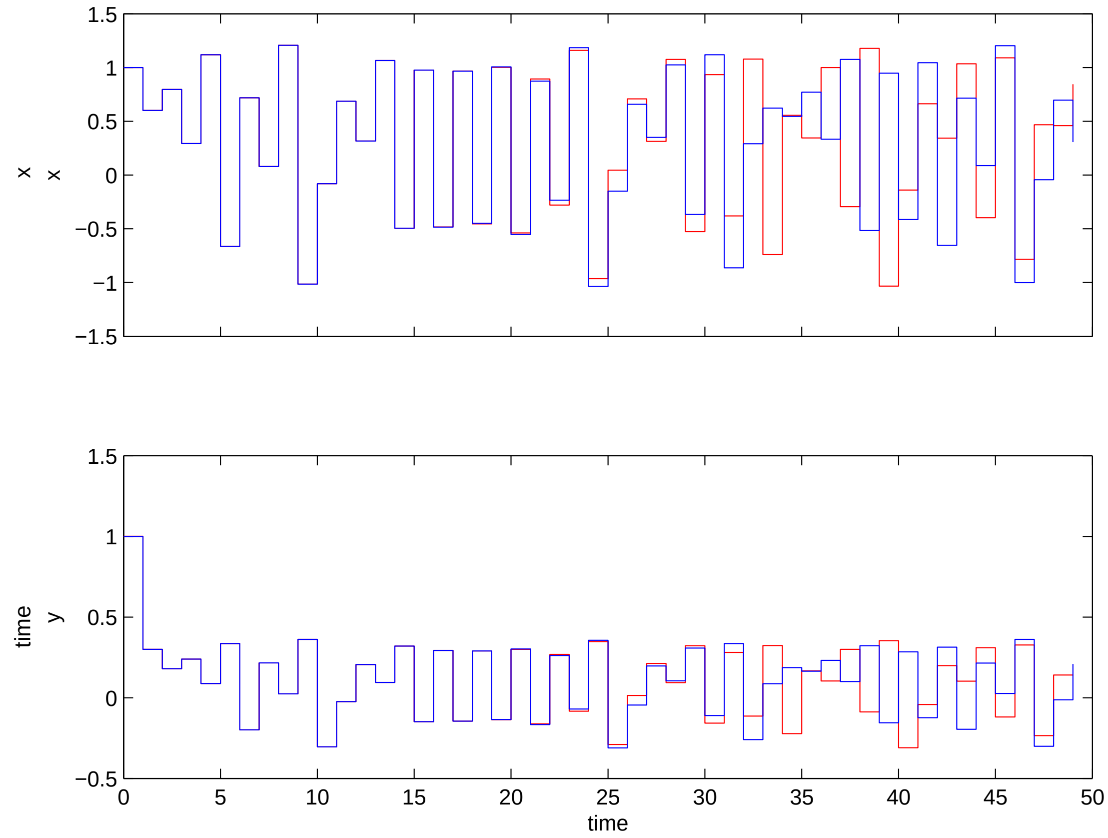
$$x_{k+1} = y_k - 1.4\, x_k^2 + 1$$
$$y_{k+1} = 0.3 x_k$$

Starting from:

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

But suppose we change the starting point *very slightly*

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1.000001 \\ 1.000001 \end{bmatrix}}_{\text{Very small } \Delta x}$$



Ways to estimate λ

1. Direct methods
    1. Lots of carefully controlled experiments
    2. Large data sets

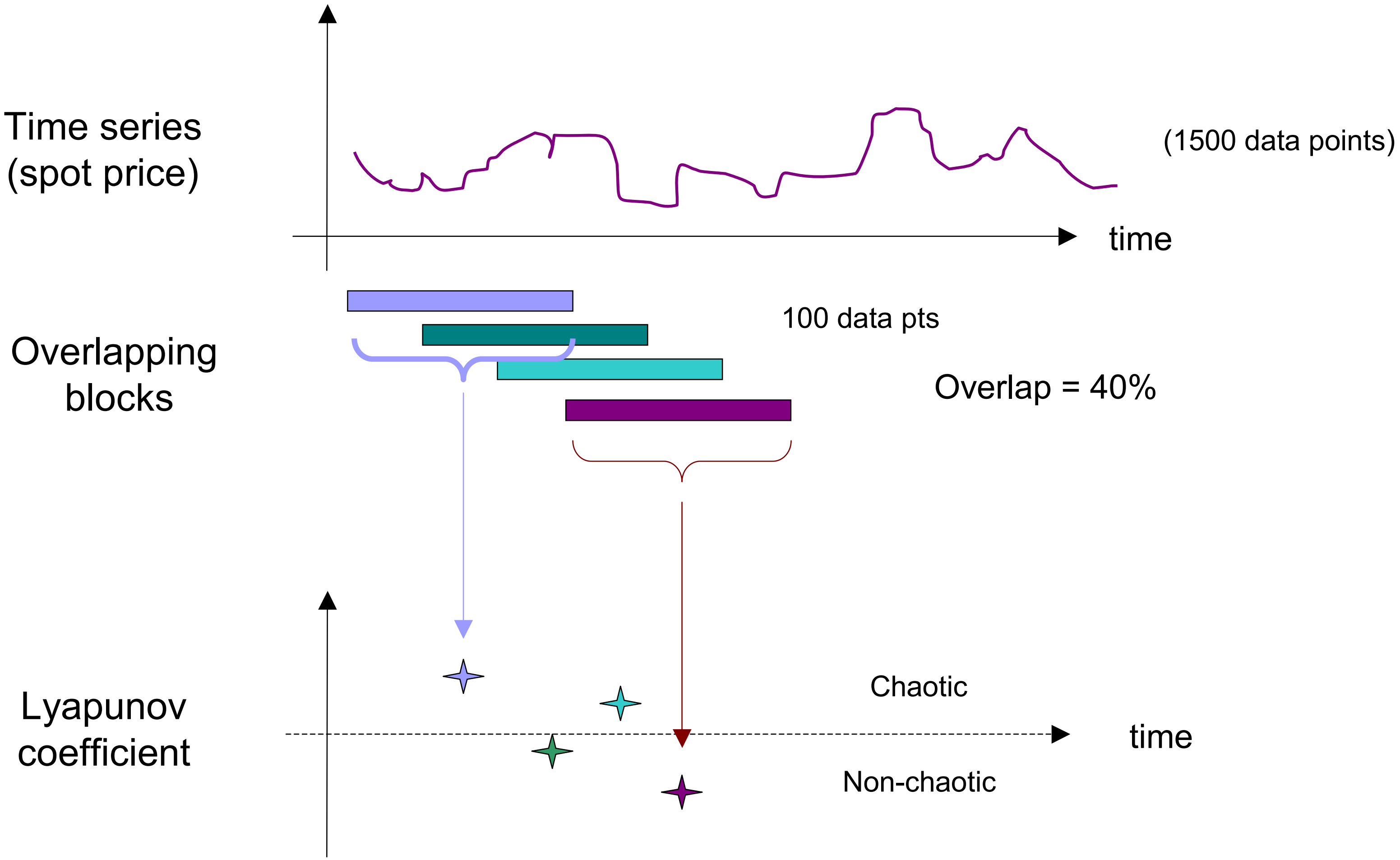
2. Jacobian methods
    1. Estimate from the individual Jacobian matrices

Finding Chaos in Noisy Systems, Nychka et al, 1992

Implementation

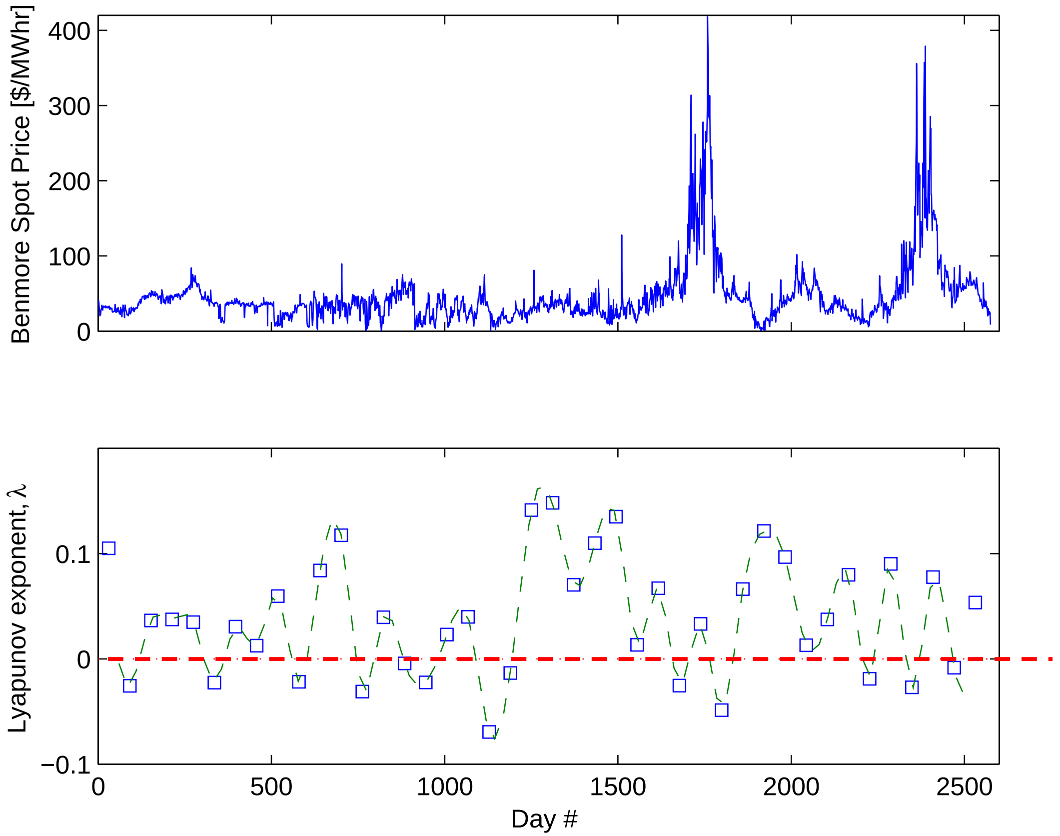
- Fortran code LENNS (from Nychka *et al*)
- Standard numerical optimisation routines (BFGS)
- Some technical modifications
- Large computation requirements
  - 50 computers @ 2 hours
  - Embarrassingly parallel
- Matlab for analysis

Weak assumptions on the stationarity of λ



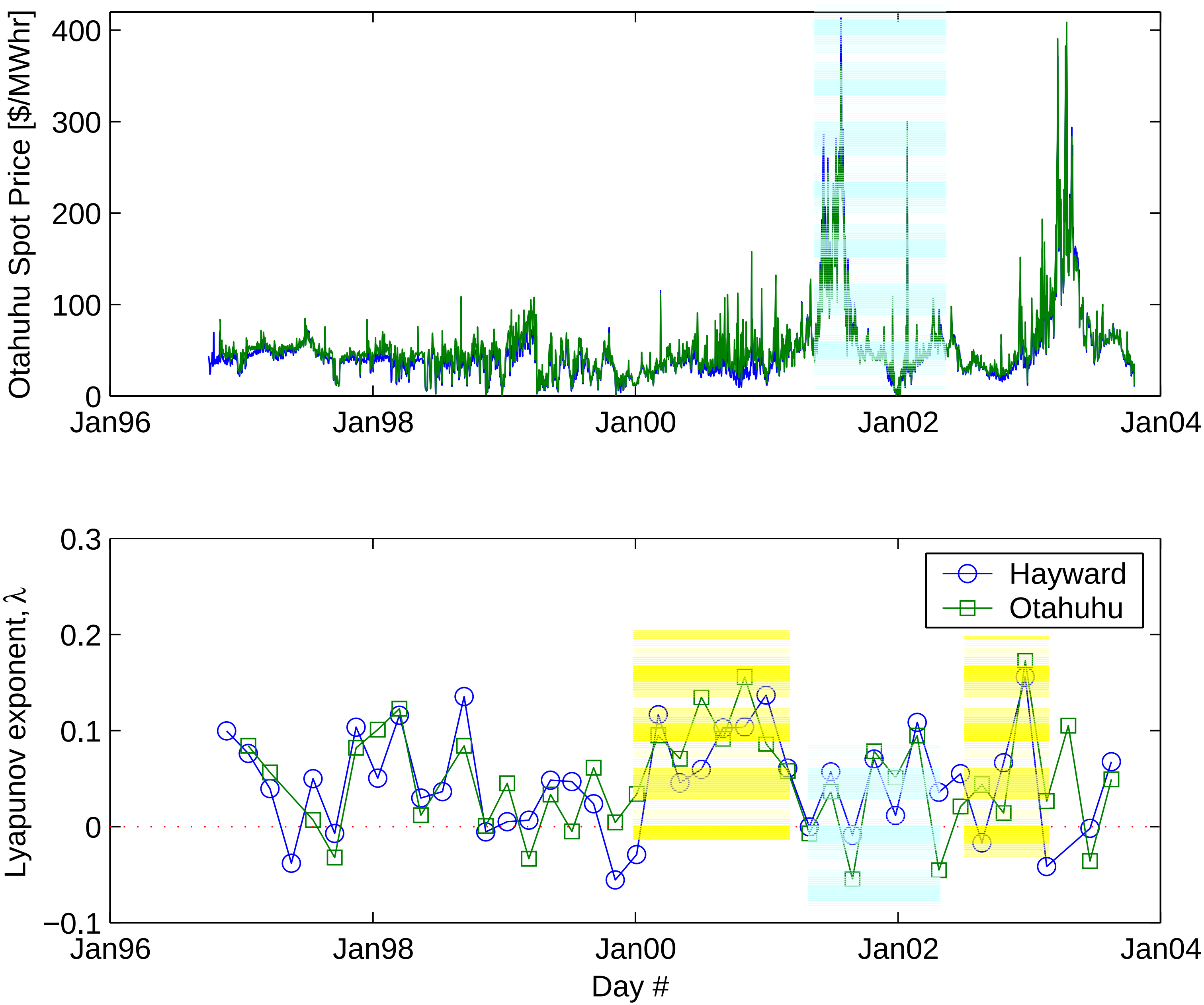
Is there any movement of λ with time?

Spot price & the dominant Lyapunov exponent



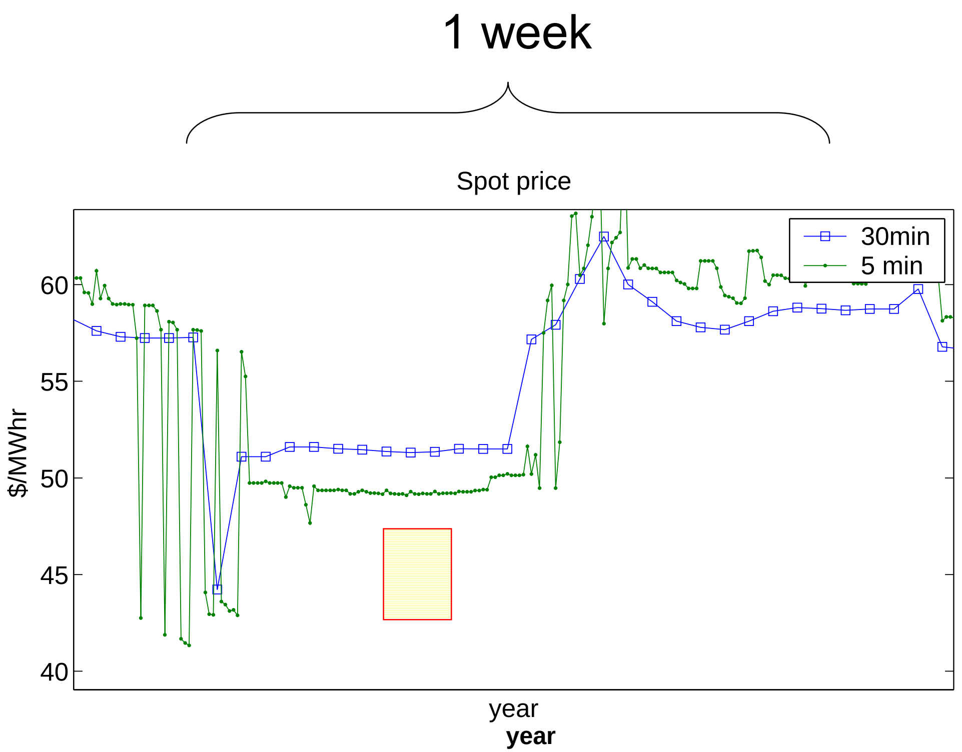
Chaotic here

Stable here


Otahuhu Spot Price [$/MWhr]
400
300
200
100
0
Jan96
Jan98
Jan00
Jan02
Jan04
Lyapunov exponent, λ
0.3
0.2
0.1
0
−0.1
Hayward
Otahuhu
Day #

Work in progress

- Relate λ to observable trends/outcomes
- Compute λ at different frequencies
  - ½ hourly price, 5 min price
  - Is λ = f(sample interval) ?



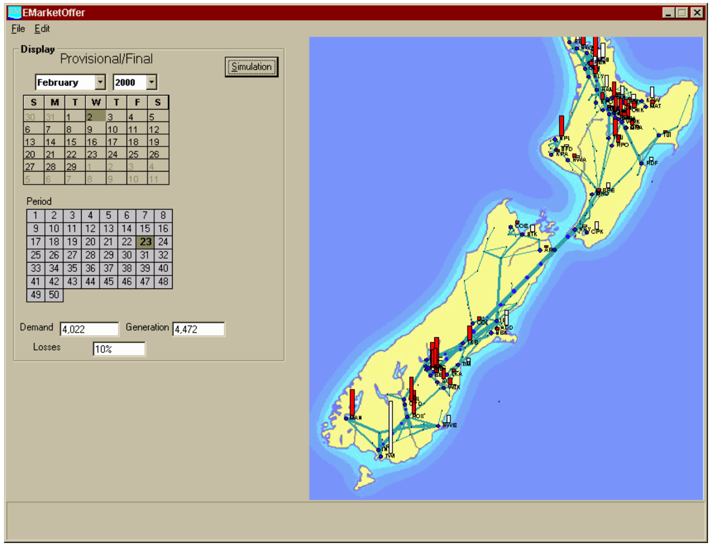
Technical issues:
1. Time delay
2. Discretisation
3. Steady-state offset ?

Consequences of chaos

- Chaotic trends are suboptimal & unsustainable

- Chaotic systems are unpredictable
  - In the medium & long term
  - Emarket http://www.energylink.co.nz/emarket.htm

- Unpredictable price increases affect industrial production.
  - *Infrastructure stocktake reportback*, M. Cullen, 2004



- The chaos may be broken by
  - Changing structure:
    - Opening feedback loops
  - Changing parameters

Conclusions

- Spot price excursions not simply correlated to climate
- Previous demand models are ….
- Spot price exhibits chaos
  - At times
  - Often before excursions
- So can we break the chaotic characteristics?

Questions ?