# Predicting the unpredictable: Is the electrical spot price chaotic?

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## ABSTRACT

Electricity necessary for our society is extremely inelastic. As a commodity, it is also essentially unstorable, and as the Maui Gas field depletes in the next few years, the supply is vulnerable, let alone sustainable.

The deregulation of the electricity supply and generation has shifted the focus of electricity modelling from long term by government agencies concentrating on supply and infrastructure, to the prediction of short term price variations by those participating in the electricity market.

However the question whether sensible predictions in the short term, (or even in the long term), are possible is relevant given the marginal success of three decades of electricity modelling both in New Zealand and around the world.

This paper compares the performance of historical models for trends in New Zealand to show both the average prediction error and that model complexity and model performance are uncorrelated. Second that paper shows that the spot price, a key output variable for the short term predictions, exhibits some chaotic characteristics. While chaotic behaviour is technically difficult to unambiguously establish given a discrete time series, the consequence makes one seriously question the advisability of expecting a predictive model to work in this situation.

Truly sustainable generating options for electricity are, we believe, not viable for New Zealand in the next decade. However what is needed as energy planners move from the resource depleting options used currently, to a more sustainable and reliable generating options in the future are models, but not models that attempt to predict chaos.

## KEY WORDS
electricity modelling, prediction, chaos, sustainable

## 1 Introduction

In the past decade, New Zealand has experience three "dry years" where the spot price for electricity increased by an order of magnitude above the base price for short periods as trended in Fig. 1. Given the extreme inelasticity of the short-term price for electricity, and the near catastrophic consequences to consumers, industry and society one hopes that these occurrences will not turn into a regular event. The economy suffered an estimated \$200 million loss [1] due to a decrease in industry production and economic activities, and the GDP growth slowed to 0.2% for 2001 September quarter compared with 2.3% the year previously, [2]. While a government initiated "Post Winter Electricity Review" received 47 submissions aimed to reduce the likelihood of a repeat, that is exactly what happened 2 years later causing a ten fold increase in spot price, and accompanying power saving campaigns. It is therefore in the nation's interest that research is done to find a better solution to the "dry year" issue.

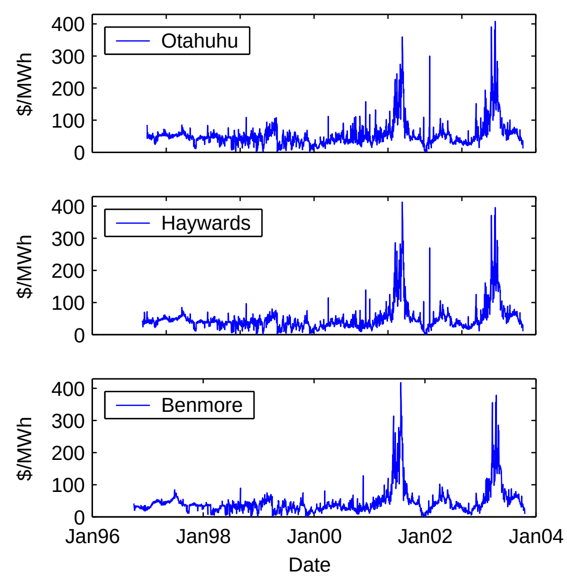



Figure 1. The spot price for electricity since deregulation in 1996. Data from [3].

While New Zealanders are used to relatively cheap electricity given the historical abundance of hydro generation capability and Maui gas, the demand is increasing by around 1.8% to 2% per year (as shown in Figures 2 and 3), Maui gas is running out, and the electricity market was deregulated in 1996. This does not bode well for the future for sustainable electricity in New Zealand.

This paper is based on the following two observations: (1) the performance of models for predicting electric power usage have been, at best modest, and (2) chaotic behaviour cannot, even in theory, be predicted over medium time ranges.

## 1.1 The importance of modelling for sustainability

A sustainable electricity supply is generally accepted to be continuous, secure and affordable that is generated without incurring adverse effects on the environment and future living standards. Clearly our present dependence on non-renewable fossil fuels (Maui gas), and the degradation to the environment from large scale hydro (notwithstanding counter arguments in say [4]), is not sustainable.

Modelling is a broad discipline that is relevant to electricity planners and policy makers. We define 3 levels of models relevant in this context: (1) Engineering/technical models of the generation hardware, (2) country-wide technical models such as those concerning transmission and climate, and (3) market models of the supply and demand culminating in a model of the spot price for electricity.

Level 1 and 2 models enable planners to weigh the various costs of generation alternatives. Due to the extreme inelasticity of electricity, it is the lack of viable generation alternatives that cause the price fluctuations exhibited in Fig. 1. Such detailed models are specific to the technology considered, and are not further discussed in this paper except to note that better models give an optimisation potential within a generating industry, and a degree of choice across industries to reduce the vulnerability to rainfall for example.

Level 3 models are concerned with the electricity market in general and the spot price in particular. The electricity market is unique in economic terms regularly exhibiting characteristics such as jump diffusion and regime switching [5]. If the supply is not secure or reliable, then it is not sustainable.

# 2 Predictive models for electricity in New Zealand

Reliable estimates for the long term future electricity demand are crucial for planning. These models span from simple blackbox models that combined smoothing and extrapolation to complex econometric models. Section 2.1 shows the modest performance of simple models, while section 2.2 shows the equally modest performance of more detailed models for New Zealand.

## 2.1 Blackbox long-term models

The oft-cited disadvantages of black-box models such as their expected poor performance in extrapolation to conditions not seen prior in the fitting stage are, to a degree, addressed in more mechanistic models such as system dynamic models or econometric models. However [6] finds little correlation between model complexity and predictive performance.

Using historical data of electricity consumption from 1943 to 1981, [7] fitted simple logistic curves of the form

$$c = F/(1 + e^{\theta_0+\theta_1 t})$$

where the saturation $F$ and scaling parameters $\theta_0, \theta_1$ were regressed. The authors then make a brave, but hedged, prediction for the long term trend of electricity consumption in 1987. The logistic model predicts a saturation around 2000 (for the total), while the alternative energy substitution model predicts a decline. With the benefit of hindsight, we now can validate these predictions as illustrated Fig. 2 and in fact logistic model under predicts the demand by around 36%.

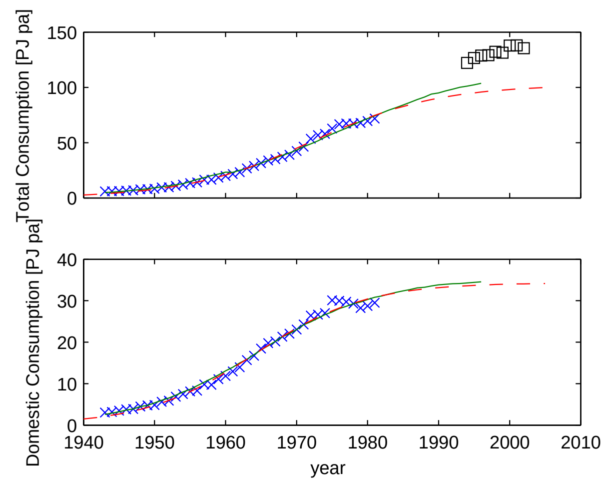



Figure 2. Total electricity consumption predictions (dashed from original paper, solid re-regressed) from [7] compared to actual data, (×). Validation data not used for the fitting is □.

## 2.2 Econometric medium-term models

Two families of medium-term econometric models from two organisations, Centre for Advanced Engineering, (CAE), and the Ministry for Economic Development (MED), are summarised in Table 1. In both cases, the basic model was subsequently updated.

The predictions for total electricity use compared to the actual data from [13] is given in Fig. 3. The prediction for hydro generation is in Fig. 4(a). The significant difference between actual and predicted is

Table 1. Predictive electricity models

| | | | |
|---|---|---|---|
| CAE | 1994, [8] | 2002, [9] | — |
| MED | 1994, [10] | 2000, [11] | 2003, [12] |

in part due to rainfall variation in volume and in the times of the year it falls.

Gas predictions are given in Fig. 4(b). The accuracy is also relatively low because of near-impossibility to predict dry years during which more gas will be used. In addition, assumptions have to be made on the rate the Maui gas field is allowed to deplete and when new gas fields will start production.

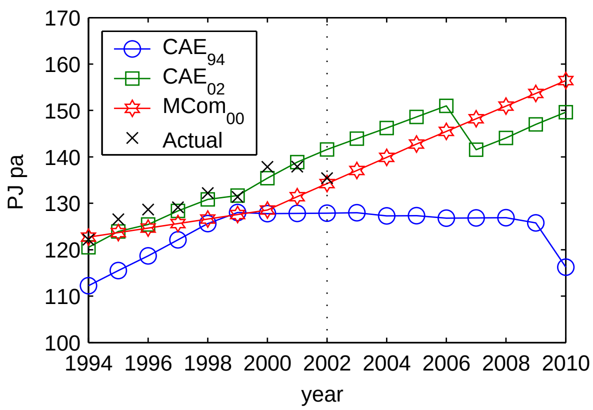


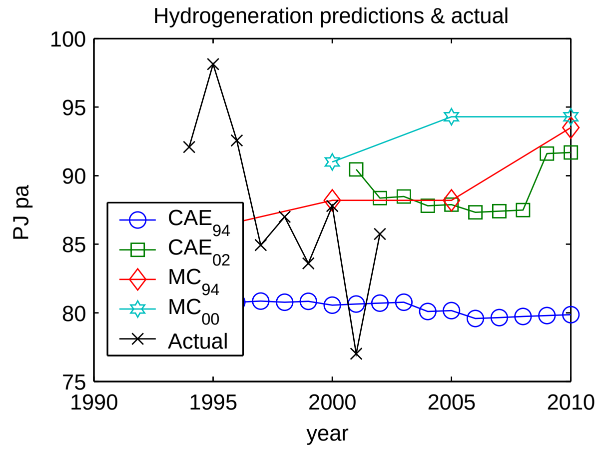

Figure 3. Total electricity use predictions compared to actual data from [8–11, 13]

So in conclusion if the complex econometric or system dynamic models fail to produce reliable predictions, or at least on a par with simple extrapolating trends, then one must question the purpose of trying to make such predictions. Similar sentiments are expressed in [14].

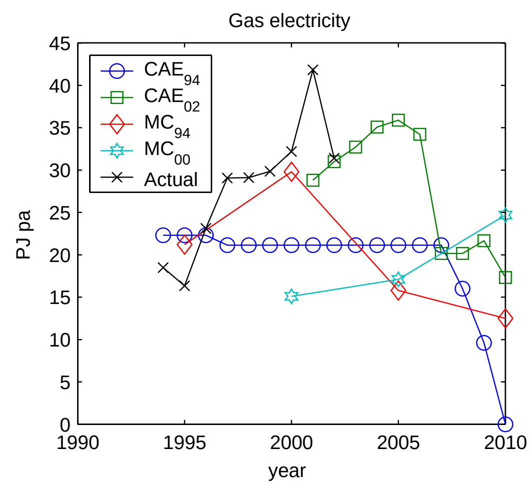
## 3 Short term chaotic models

Given the deregulation of the electricity industry, the modelling holy grail is to predict the spot price. The question is not whether this prediction is reasonable, but if it is even theoretically possible. Cases where the prediction is likely to be unsuccessful are where the underlying time series is random or chaotic.

One of best indicators of chaos is the dominant Lyapunov exponent, $\lambda$. For a bounded system, a positive $\lambda$ implies that neigbouring trajectories exponentially diverge, leading to chaos. While it is one thing to reliably compute Lyapunov exponents for deterministic dynamic systems using numerical experiments, it is quite another to extract $\lambda$ from an experimental time series, possibly corrupted with a stochastic input. For




(a) Hydro generation predictions




(b) Gas generation predictions

Figure 4. Model predictions for hydro and gas. Data from [8–11, 13].

example [15] concludes that the natural gas futures data are nonlinear but not chaotic, while [16] concludes that the natural gas liquid markets are chaotic.

The problems are of a practical nature. To adequately estimate indicators of chaotic behaviour such as Lyapunov exponents, embedded dimensions etc, we need a large amount of regularly spaced time series data. The freely available public domain data are: spot price, reservoir levels.

However restricting only to a data-driven approach means that we must identify this from a single one-dimensional time series of data. However the embedding theorem, if applicable, means that this single time series contains all the necessary information to establish chaos, [17].

One estimate of the dominant Lyapunov exponent given $N$ values is

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \log \|\hat{\boldsymbol{T}}_N\|$$

where the matrix $\boldsymbol{T}_N = \hat{\boldsymbol{J}}_N \cdot \hat{\boldsymbol{J}}_{N-1} \cdots \hat{\boldsymbol{J}}_1$ is built from a series of Jacobian matrices of the underlying nonlinear map. As in this case the underlying nonlinear function is unknown, [18] suggest using neural network models as they appear to be less sensitive to noise (compared to direct methods), and are well suited to capture the nonlinear behavior inherent in chaotic systems. This work uses the LENNS (Lyapunov Exponent of Noisy Nonlinear Systems) routines adapted from [19].

Fig. 5 trends the dominant Lyapunov exponent over time for the spot price at Benmore. Benmore was deliberately chosen as it is closest to a significant generating station, and therefore less influenced by outside factors such as transmission limitations. Nonetheless, at times the estimated Lyapunov exponent is positive, indicating that the time series exhibits chaos. The trend to chaos is, not surprisingly, highly correlated to the periods where there was intense activity in the electricity market.

The fact that the spot price is sporadically chaotic has some significant consequences. The first is that chaotic trends are unpredictable in the medium and long terms, although some argue workable predictions are possible in the short term. (The magnitude of 'short' depends on $\lambda$.) The second consequence is that the chaos, which adversely affects the market, needs no significant external input; stochastic, chaotic or otherwise. It is often simply a non trivial function of the underlying nonlinearities in the dynamics, and the current parameters. This means that future chaos in the electricity market need not be triggered by concerns in early Autumn regarding lake levels, or that the triggering concern is disproportionately small.

However we should note that the computation to estimate $\lambda$ is a delicate one, and one that requires considerable computing resources. We repeat that $\hat{\lambda}$ is an estimate obtained by looping over model predictions derived using different values of embedding dimension, delays, and different number of parameters in a neural network model. Furthermore, due to the nature of the computation, it is difficult to derive reliable uncertainty estimates for $\lambda$.

## 4 Conclusions

It has long been accepted within the electricity industry that improved modelling helps to avoid future power crises. This paper identifies 3 levels of models and argues that the quality of predictions is modest for all models reviewed, and that chaos was observed in the spot price.

A sustainable electricity supply is a secure one where the spot price is constant and predictable as opposed to what has happened recently in New Zealand. Large excursions in spot price disrupt the economy, and encourage the consuming of non-renewable resources to generate power during the crises. The generally accepted reasons for the recent periods of elevated spot prices have been attributed to the concern in early autumn about the low levels in the hydro reservoirs. However there is in fact another interpretation based on the fact that the spot price market is chaotic. The nature of chaotic systems is a delicate function of their parameters. Minor changes in model parameters, or subtle changes in model structure can flip a system into, and out of, chaos. The key point is that the chaos can be triggered by disproportionately small events, either in structure such as an addition of an extra feedback loop, or in parameters, or even in operating point.

Clearly the prediction of the demand and pricing of electricity is challenging. In the review of long term energy forecasting in the US, [14] acknowledge that the reasons for forecasting was not in fact to forecast energy usage, (that was not deemed successful), but for secondary purposes such as to help energy policy planners judge the consequences of action or inaction. In a similar manner, we suggest that while expecting useful medium term predictions from chaotic systems is questionable, the study of them is not. For example if the mechanisms are known that trigger the chaos, then these may be circumvented for example by operating practice or legislation, and we avoid the excessive outbursts in spot price that if left alone will only mean that electricity in New Zealand is unsustainable.

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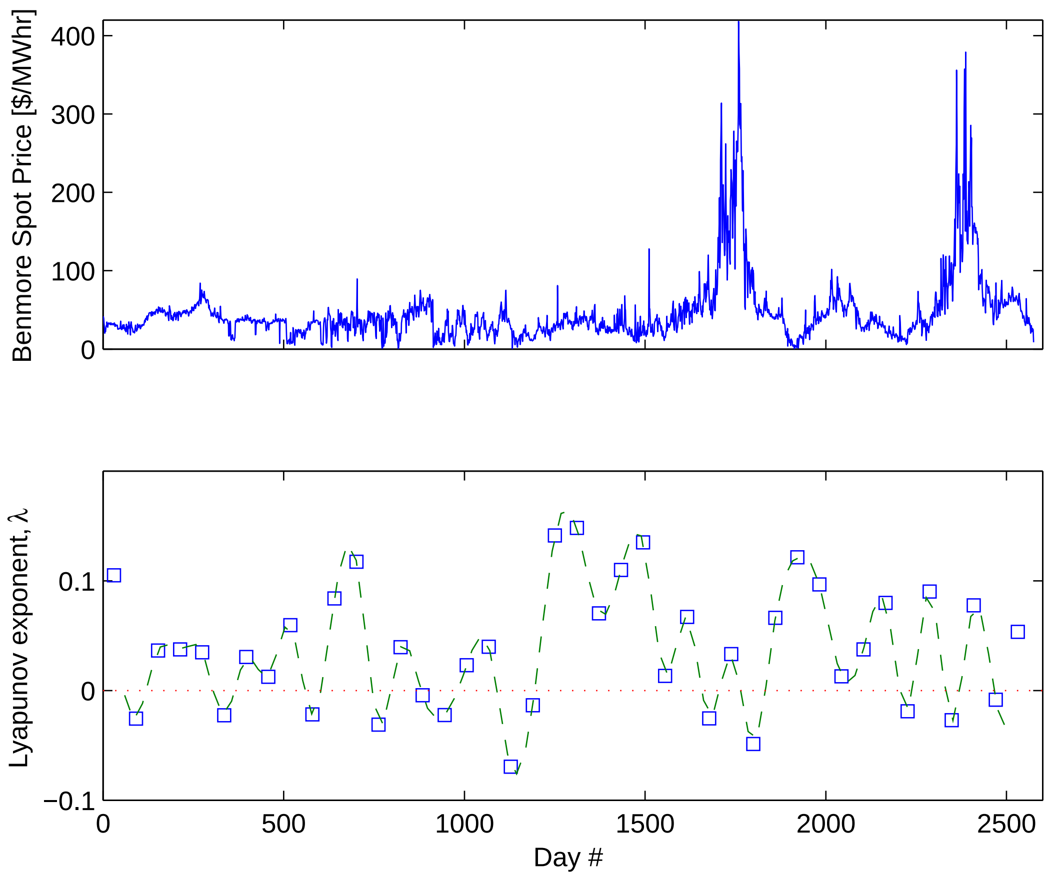


Figure 5. An estimate of the dominant Lyapunov exponent, $\hat{\lambda}$, as a function of time for the Benmore spot price.

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